INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2016-17
Statistics - II, Semesteral Examination, May 4, 2017
Answer all questions. **Time: 3 Hours**
You may use any result stated in the class by stating it.

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(100, \sigma^2)$ where σ^2 is unknown. Consider testing at level α,

$$H_0 : \sigma^2 \leq 20 \text{ versus } H_1 : \sigma^2 > 20.$$

(a) Show that the conditions required for the existence of UMP test are satisfied here.
(b) Derive UMP test of level α.
(c) Consider the test which rejects H_0 whenever $\sum_{i=1}^n (X_i - \bar{X})^2 > C$ where $C > 0$ is such that $\sup_{\sigma^2 \leq 20} P(\sum_{i=1}^n (X_i - \bar{X})^2 > C) = \alpha$. Show that this test is not UMP test of level α. **[10]**

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with density $f(x|\lambda) = \lambda \exp(-\lambda x)$, $x > 0$, where $\lambda > 0$ is unknown. For testing

$$H_0 : \lambda = 1 \text{ versus } H_1 : \lambda \neq 1,$$

find the generalized likelihood ratio test at the significance level α. **[8]**

3. Let $X_1, X_2, \ldots, X_n$ be i.i.d. $N(\mu, \sigma^2)$, where $\mu \geq 0$ and $\sigma^2 > 0$. Let $\theta = (\mu, \sigma^2)$.
(a) What is the parameter space Θ in this model?
(b) Find the m.l.e., $(\hat{\mu}, \hat{\sigma}^2)$, of (μ, σ^2).
(c) Find the UMVUE, $\hat{\mu}^*$, of μ.
(d) Show that $E((\hat{\mu} - \mu)^2) \leq E((\hat{\mu}^* - \mu)^2)$ for all $\theta \in \Theta$. **[11]**

4. The weekly number of fires, X, in a town has the Poisson(θ) distribution. The numbers of fires observed for five weekly periods were 0, 1, 1, 0, 0. Assume that these observations are independent, and that the prior distribution on θ is $\pi(\theta) \propto \theta \exp(-10\theta) I_{(0,\infty)}(\theta)$.
(a) Derive the posterior distribution θ given the data.
(b) Find the posterior mean and posterior standard deviation of θ. **[10]**

5. Let $X_1, X_2, \ldots, X_n$ be i.i.d. Poisson(λ), $\lambda > 0$, and let $Y_i = 1$ when $X_i > 0$, and 0 otherwise, $i = 1, 2, \ldots, n$.
(a) Show that $\bar{X}$ is a consistent estimator of λ, and it is asymptotically normally distributed.
(b) Find a transform, $g(\bar{Y})$, of $\bar{Y}$ which is a consistent estimator of λ; derive its asymptotic distribution.
(c) Compare the asymptotic relative efficiency of $\bar{X}$ with respect to $g(\bar{Y})$. **[11]**